Minimization of DFA

If (p, q) are equivalent then

$$\delta(p, w) \in F \Rightarrow \delta(q, w) \in F$$

or

$$\delta(p, w) \notin F \Rightarrow \delta(q, w) \notin F$$

The equivalent is depends on length of input string.

$|w| = 0$ that is 0-equivalent

$Q = \{\rightarrow q_0, q_1, q_2, {}^{*}q_3\}$

$\{q_0, q_1, q_2\}$ $\{q_3\}$ ← 0 equivalent

↑ no final states ↑ final state.

means without taking any input, we can distinguish states into two groups.

$|w| = 1$, 1-equivalent.

for ex^{n} (p, q) $\Sigma = \{0, 1\}$

$\delta(p, 0) =$ $\delta(p, 1) =$

$\delta(q, 0) =$ $\delta(q, 1) =$

$|w| = 2$, 2-equivalent

$|w| = 3$, 3-equivalent.

Partition Method

→ Delete all states that are not reachable from start / initial state.

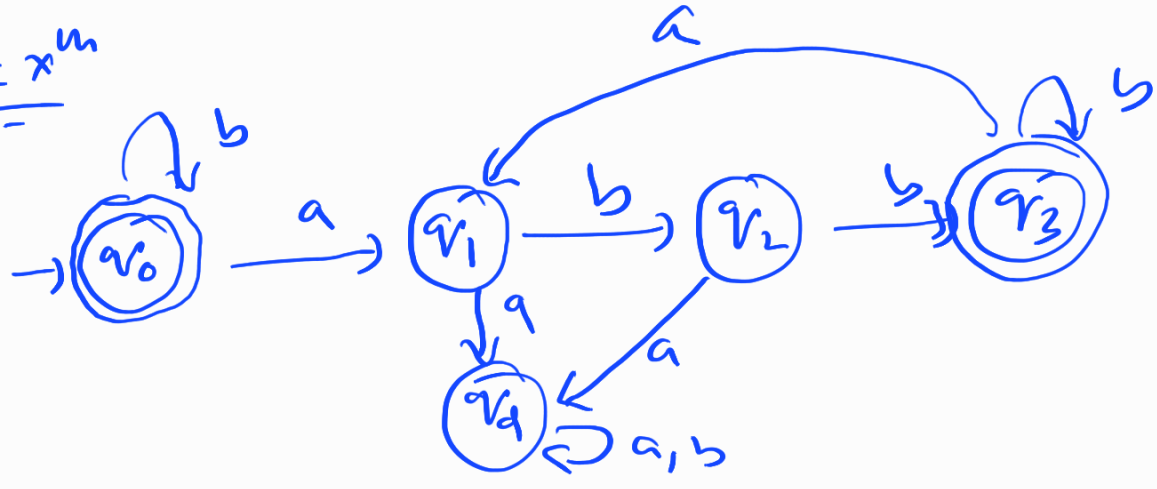
→ Initialize $k = 1$

→ Find (P_k) by partitioning the sets of (P_{k-1})
If two states of a set are distinguishable, we will split the set into different set in P_k.

→ Stop, when $P_k = P_{k-1}$

→ Merge all states of one set.

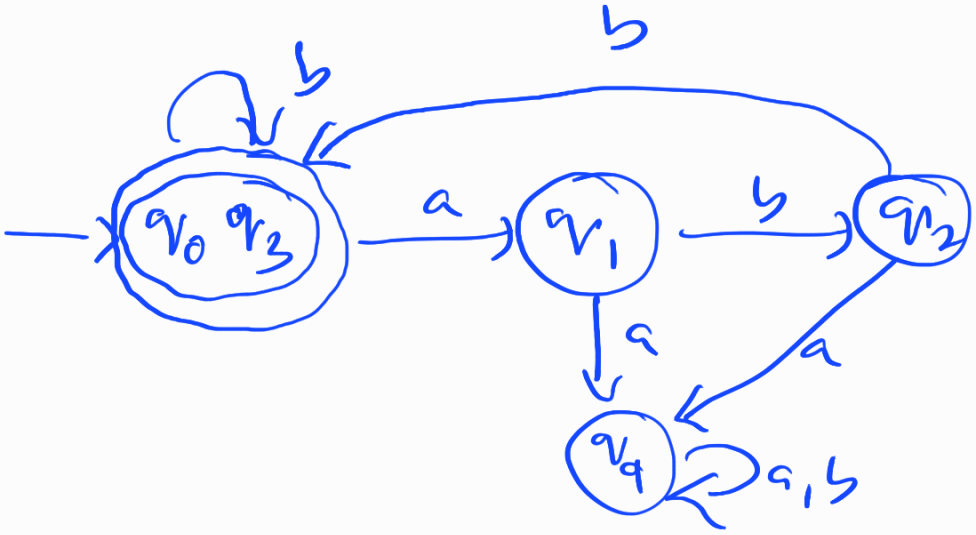
Exm



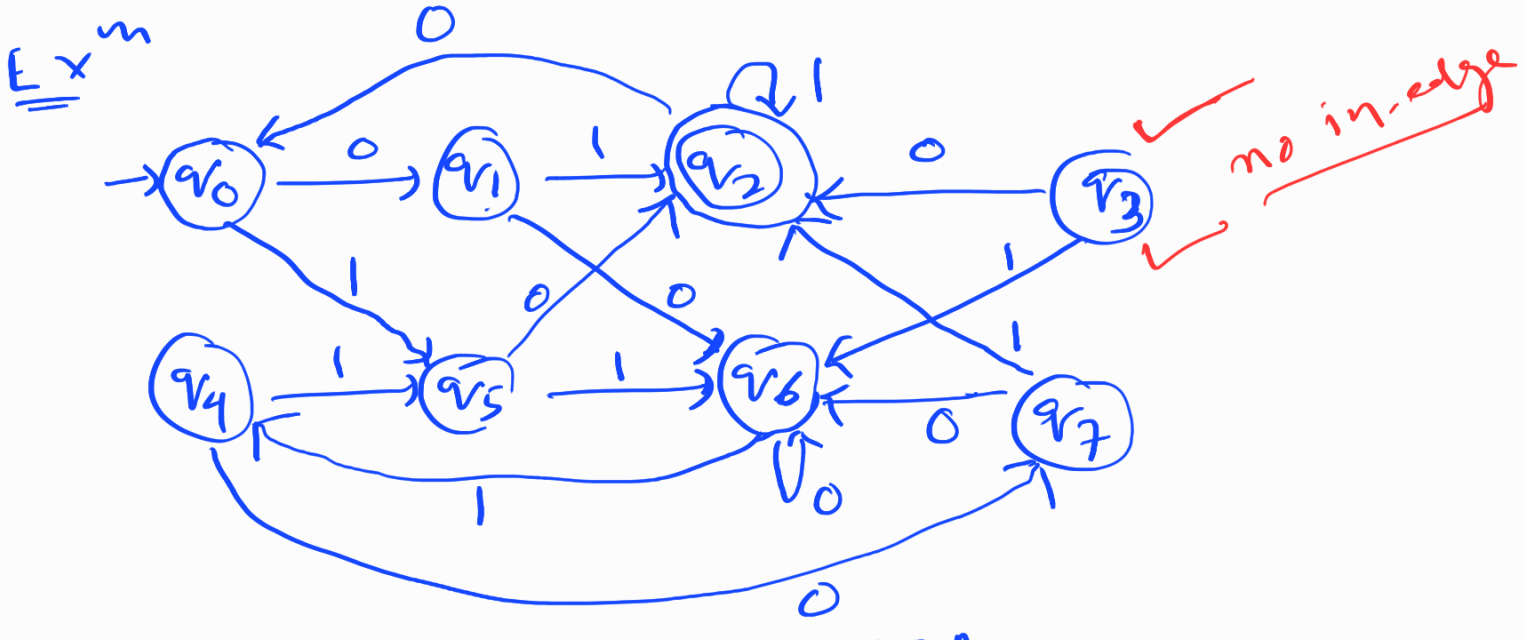
	a	b
→*q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_4	q_3
q_3	q_1	q_3
q_4	q_4	q_4

	non-final	final
0-equivalent	$\{q_1, q_2, q_4\}$	$\{q_0, q_3\}$
1-equivalent	$\{q_1, q_4\}\{q_2\}$	$\{q_0, q_3\}$
2-equivalent	$\{q_1\}\{q_4\}\{q_2\}$	$\{q_0, q_3\}$
3-equivalent	$\{q_1\}\{q_1\}\{q_2\}$	$\{q_0, q_3\}$

merge



Ex^n



minimize the above DFA

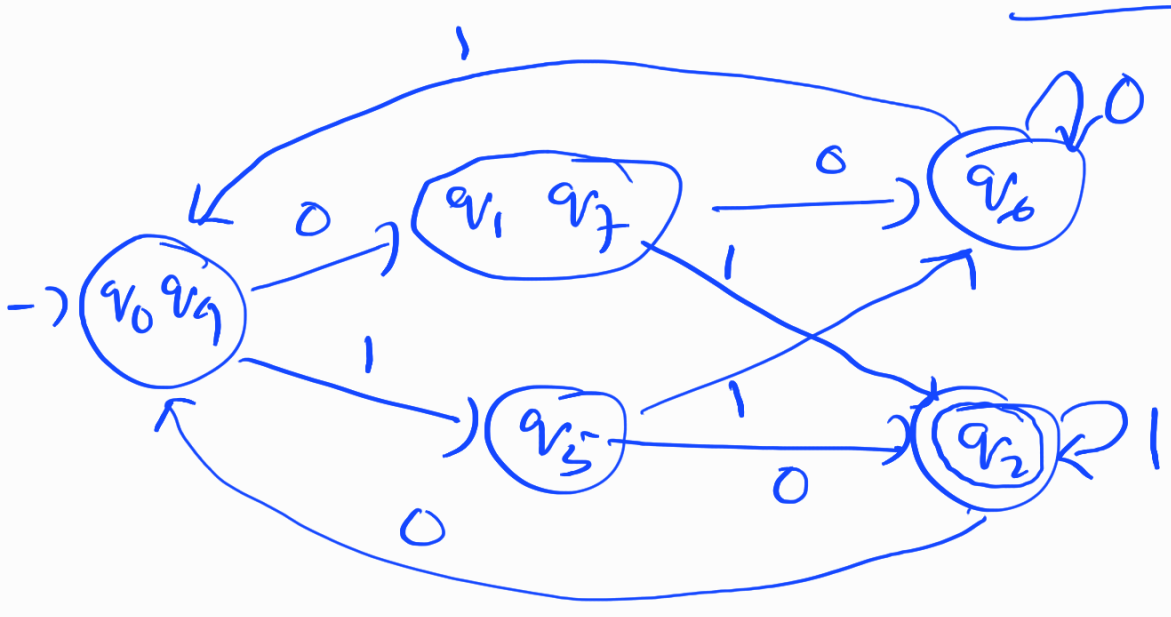
	0	1	
$\to q_0$	q_1	q_5	
q_1	q_6	q_2	
$*q_2$	q_0	q_2	
q_3	q_2	q_6	← unreachable
q_4	q_7	q_5	
q_5	q_2	q_6	
q_6	q_6	q_4	
q_7	q_6	q_2	

0- equivalent $\{q_0, q_1, q_4, q_5, q_6, q_7\}\ \{q_2\}$

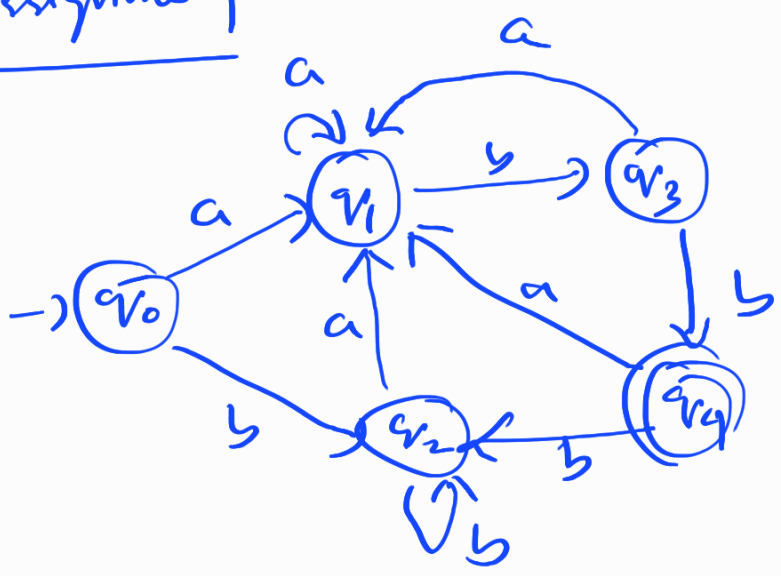
1- equivalent $\{q_0, q_4, q_6\}\ \{q_1, q_7\}\{q_5\}\ \{q_2\}$

2- equivalent $\{q_0, q_4\}\ \{q_6\}\ \{q_1\ q_7\}\ \{q_5\}\ \{q_2\}$

3- equivalent $\{q_0, q_4\}\ \{q_6\}\ \{q_1, q_7\}\ \{q_5\}\ \{q_2\}$



Assignment



Minimize the DFA.

	a	b
→ q_0	q_1	q_2
q_1	q_1	q_3
q_2	q_1	q_2
q_3	q_1	q_4
* q_4	q_1	q_2

0- equivalent $\{q_0, q_1, q_2, q_3\}$ $\{q_4\}$

1- equivalent $\{q_0, q_1, q_2\}$ $\{q_3\}$ $\{q_4\}$

2- equivalent $\{q_0, q_2\}$ $\{q_1\}$ $\{q_3\}$ $\{q_4\}$

3- equivalent $\{q_0, q_2\}$ $\{q_1\}$ $\{q_3\}$ $\{q_4\}$

Merge