**M.Tech(CSE) 1st Semester Examination -2019**
**Subject: Mathematical Foundations of Computer Science (MFCS)**

***Time: 3 Hours*** ***Marks: 70***

***Answer all questions.***
***The figure in the right hand margin indicates marks.***

**Q1** a) What is the negation and contrapositive of the following implication: **[3]**
If capital investment remains unchanged, then government spending will increase or unemployment will result.

b) Show that $((p \vee q) \wedge \sim(\sim p \wedge (\sim q \vee \sim r))) \vee (\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)$ is a tautology. **[5]**

c) Show that $t$ is a valid conclusion from the premises $p \Rightarrow q, q \Rightarrow r, r \Rightarrow s, \sim s\ and\ p \vee t$. **[6]**

**<u>OR</u>**

a) Find a formula $A$ that uses the variables $p, q$ and $r$ such that $A$ is a contradiction. **[3]**

b) Show that if $a_n$ is defined by $a_1 = 1, a_2 = 5$ and $a_{n+1} = 5a_n - 6a_{n-1}$ for $n \geq 2$, then $a_n = 3^n - 2^n$ for all $n \in N$. **[5]**

c) Prove or disprove the validity of the following arguments: **[6]**

*Every living thing is a plant or an animal.*
*David's dog is alive and it is not a plant.*
*All animals have hearts.*
___
∴ *David's dog has a heart.*

**Q2** a) What do you mean by reflexive closure and symmetric closure of a relation? Find the reflexive closure and symmetric closure of the relation $R = \{(1,1),(2,2),\ (2,5),(3,4\ ),(4,3),(4,5)\}$ on $A = \{1,2,3,4,5\}$ **[3]**

b) Solve the following recurrence relation: **[5]**

$$a_n = 4a_{n-1} + 5a_{n-2}\ \forall n \geq 3\ with\ a_1 = 2\ and\ a_2 = 6.$$

c) Find the transitive closure of $R$ by using Warshall's algorithm where $R$ be a relation on set $A = \{1,2,3,4,5\}$ defined by $a\ R\ b$ if and only if $|a - b| = 2$ **[6]**

**<u>OR</u>**

a) Let $R$ be an equivalence relation on set of integers Z defined by **[3]**

$$R = \{(x,y): x \in Z, y \in Z\ and\ x \equiv y\ mod\ 3\}.$$

Find the partition of $Z$ corresponding to $R$.

b) If $R$ is the relation on the set of ordered pairs of positive integers such that $(a,b),(c,d) \in R$ whenever $ad = bc$, show that $R$ is an equivalence relation. **[5]**

c) Draw the Hasse diagram of the Poset $[A; |]$ where $A = \{2,3,4,6,8,24,48\}$. Also determine: **[6]**

i) The Maximal and minimal elements of A.
ii) The least and greatest elements of A.
iii) The upper bounds and LUB of 2 and 4.
iv) The lower bounds and GLB of 24 and 48.

**Q3** a) Prove that the identity element (if it exists) of any algebraic structure is unique. **[3]**

b) In any Boolean algebra B, for all a. b $\in$ B **[5]**

Prove that a ∨ (a ∧ b) = a and a ∧ (a ∨ b) = a.

c) Prove that the necessary and sufficient condition for a non empty sub-set $H$ of a group $(G,*)$ to be a sub group is $a \in H, b \in H \Rightarrow a * b^{-1} \in H,$ where $b^{-1}$ is the inverse of $b$ in $G$. **[6]**

**OR**

a) Show that the set $\{1, 2, 3, 4, 5\}$ is not a group under addition modulo 6. **[3]**

b) If $R$ is the set of real numbers and $*$ is the operation defined by $a * b = a + b + 3ab$, where $a, b \in R$, show that $[R, *]$ is a commutative monoid. Which elements have inverses and what are they? **[5]**

c) Let $(L, \preccurlyeq)$ be a lattice. Then for $a, b \in L$ show that **[6]**

i) $a \vee b = b \; iff \; a \preccurlyeq b$

ii) $a \wedge b = a \; iff \; a \preccurlyeq b$

iii) $a \wedge b = a \; iff \; a \vee b = b$

**Q4** a) For a set of 10 multiple choice questions, where each question has four options, find the number of ways of answering all questions. **[3]**

b) Prove by Pigeonhole principle that if seven integers from 1 to 12 are chosen, then two of them will add up to 13. **[5]**

c) A company purchased 100,000 transistors: 50, 000 from supplier A, 30, 000 from supplier B, and 20, 000 from supplier C. It is known that 2 percent from supplier A's transistors are defective, 3 percent of supplier B's transistors are defective, and 5 percent of supplier C's transistors are defective. Given that a transistor selected at random is defective, what is the probability that it is from supplier B? **[6]**

**OR**

a) The probabilities of A, B C solving a problem are 1/3, 2/7, 3/8 respectively. If all they try to solve the problem simultaneously, what is the probability that the problem will be solved? **[3]**

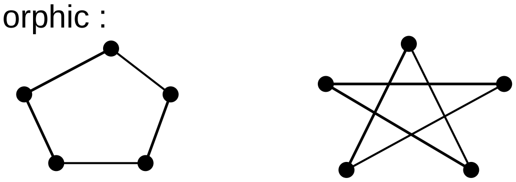
b) A continuous random variable has the following density function: **[5]**

$$f(x) = \begin{cases} \frac{1}{2} - ax & 0 \leq x \leq 4 \\ 0 & otherwise \end{cases}$$

Find the value of a and then compute $P(1 < x < 2)$.

c) A man has 7 relatives, 4 of them are ladies and 3 gentlemen, his wife has 7 relatives and 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relatives and 3 of wife's relatives? **[6]**

**Q5** a) Define isomorphism. Determine whether the following pair of graphs are isomorphic : **[3]**



b) How many vertices do the following graphs have if they contain? **[5]**

i) 16 edges and all vertices of degree 2.

ii) 21 edges, 3 vertices of degree 4 and others each of degree 3.

c) Apply the Havel-Hakimi result to determine if the following degree sequences are graphic. If so draw such graph. **[6]**

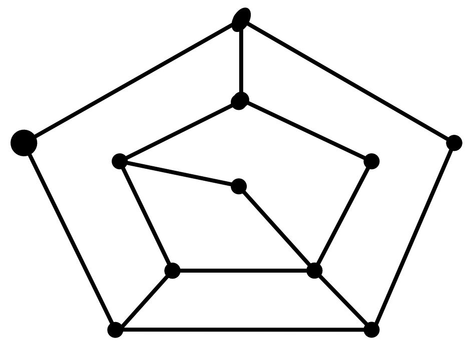
i) (1, 1, 1, 2, 2, 2, 3, 3, 4, 7)

ii) (1, 3, 3, 4, 5, 5, 5, 5, 5)

**OR**

a) Prove that if a graph $G$ has no loops or multiple edges, then number of vertices of odd degree is an even number. **[3]**

b) Determine whether the following graph is Hamiltonian. Justify your answer. **[5]**



c) Write Fleury's algorithm and then using the algorithm find Euler's circuit of the following graph: **[6]**