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| **I-M.Tech(CSE)-MFCS(1.1)** | | | | | | |
| **M.Tech(CSE) 1st Semester Examination -2019** | | | | | | |
| **Subject: Mathematical Foundations of Computer Science (MFCS)** | | | | |  | |
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| ***Time: 3 Hours*** | | | | ***Marks: 70*** | | |
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| ***Answer all questions.*** | | | | | | |
| ***The figure in the right hand margin indicates marks.*** | | | | | | |
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| **Q1** | a) | What is the negation and contrapositive of the following implication:  If capital investment remains unchanged, then government spending will increase or unemployment will result. | | | | **[3]** |
|  | b) | Show that is a tautology. | | | | **[5]** |
|  | c) | Show that *t* is a valid conclusion from the premises. | | | | **[6]** |
|  |  | **OR** | | | |  |
|  | a) | Find a formula that uses the variables and such that is a contradiction. | | | | **[3]** |
|  | b) | Show that if is defined by and for , then for all . | | | | **[5]** |
|  | c) | Prove or disprove the validity of the following arguments:   |  | | --- | | *Every living thing is a plant or an animal.* | | *David’s dog is alive and it is not a plant.* | | *All animals have hearts.* | | *David’s dog has a heart.* | | | | | **[6]** |
|  |  |  | | | |  |
| **Q2** | a) | What do you mean by reflexive closure and symmetric closure of a relation? Find the reflexive closure and symmetric closure of the relation on | | | | **[3]** |
|  | b) | Solve the following recurrence relation: | | | | **[5]** |
|  | c) | Find the transitive closure of by using Warshall's algorithm where be a relation on set defined by if and only if | | | | **[6]** |
|  |  | **OR** | | | |  |
|  | a) | Let be an equivalence relation on set of integers defined by  Find the partition of corresponding to . | | | | **[3]** |
|  | b) | If is the relation on the set of ordered pairs of positive integers such that whenever , show that is an equivalence relation. | | | | **[5]** |
|  | c) | Draw the Hasse diagram of the Poset where  Also determine:   1. The Maximal and minimal elements of A. 2. The least and greatest elements of A. 3. The upper bounds and LUB of 2 and 4. 4. The lower bounds and GLB of 24 and 48. | | | | **[6]** |
|  |  |  | | | |  |
| **Q3** | a) | Prove that the identity element (if it exists) of any algebraic structure is unique. | | | | **[3]** |
|  | b) | In any Boolean algebra B, for all a. b ∈ B  Prove that a ∨ (a ∧ b) = a and a ∧ (a ∨ b) = a. | | | | **[5]** |
|  | c) | Prove that the necessary and sufficient condition for a non empty sub-set of a group to be a sub group is where is the inverse of in . | | | | **[6]** |
|  |  | **OR** | | | |  |
|  | a) | Show that the set is not a group under addition modulo 6. | | | | **[3]** |
|  | b) | If is the set of real numbers and is the operation defined by , where , show that is a commutative monoid. Which elements have inverses and what are they? | | | | **[5]** |
|  | c) | Let be a lattice. Then for show that | | | | **[6]** |
|  |  |  | | | |  |
| **Q4** | a) | For a set of 10 multiple choice questions, where each question has four options, find the number of ways of answering all questions. | | | | **[3]** |
|  | b) | Prove by Pigeonhole principle that if seven integers from 1 to 12 are chosen, then two of them will add up to 13. | | | | **[5]** |
|  | c) | A company purchased 100,000 transistors: 50, 000 from supplier A, 30, 000 from supplier B, and 20, 000 from supplier C. It is known that 2 percent from supplier A’s transistors are defective, 3 percent of supplier B’s transistors are defective, and 5 percent of supplier C’s transistors are defective. Given that a transistor selected at random is defective, what is the probability that it is from supplier B? | | | | **[6]** |
|  |  | **OR** | | | |  |
|  | a) | The probabilities of A, B C solving a problem are 1/3, 2/7, 3/8 respectively. If all they try to solve the problem simultaneously, what is the probability that the problem will be solved? | | | | **[3]** |
|  | b) | A continuous random variable has the following density function:  Find the value of a and then compute . | | | | **[5]** |
|  | c) | A man has 7 relatives, 4 of them are ladies and 3 gentlemen, his wife has 7 relatives and 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man’s relatives and 3 of wife’s relatives? | | | | **[6]** |
|  |  |  | | | |  |
| **Q5** | a) | Define isomorphism. Determine whether the following pair of graphs are isomorphic : | | | | **[3]** |
|  | b) | How many vertices do the following graphs have if they contain?   1. 16 edges and all vertices of degree 2. 2. 21 edges, 3 vertices of degree 4 and others each of degree 3. | | | | **[5]** |
|  | c) | Apply the Havel-Hakimi result to determine if the following degree sequences are graphic. If so draw such graph.   1. (1, 1, 1, 2, 2, 2, 3, 3, 4, 7) 2. (1, 3, 3, 4, 5, 5, 5, 5, 5) | | | | **[6]** |
|  | **OR** | | | | |  |
|  | a) | Prove that if a graph has no loops or multiple edges, then number of vertices of odd degree is an even number. | | | | **[3]** |
|  | b) | Determine whether the following graph is Hamiltonian. Justify your answer. | | | | **[5]** |
|  | c)  a  d  b  c  e  f  g  h  i  j | Write Fleury’s algorithm and then using the algorithm find Euler’s circuit of the following graph: | | | | **[6]** |