SEMESTER EXAMINATION– 2019

SUBJECT: Combinatorics & Graph Theory

BRANCH(S)/PROGRAMME: Int. MCA SEMESTER: 7th

Answer Question No.1 which is compulsory and any FIVE from the rest.

Time: 3 Hour **(The figures in the right hand margin indicate marks.)** **Total marks: 70**

1. Answer all. [2x10=20]

   (a) Define a finite graph. Give an example of infinite graph.

   (b) If two graphs G_1 and G_2 are edge disjoint, what is their ring sum?

   (c) Define centre of a tree and find the centre of a tree drawn by you.

   (d) What is rank and nullity of a graph ?

   (e) Give necessary and sufficient condition for a graph to be Euler's graph.

   (f) What is a fundamental circuit ?

   (g) Give two Kuratowski's graph and mention two properties of these graphs.

   (h) What is the dimension of a vector space of a graph ?

   (i) If a graph G is separable and consist of two blocks g1 and g2, the incidence matrix A(G) of graph G can be presented as __________________ .

   (j) How many triangles can be formed by 8 points of which 3 are collinear?

2. (a) Prove that the number of vertices of odd degree in a graph is always even. [5]

   (b) Prove that a simple graph with *n* vertices and *k* components can have at most $(n-k)(n-k+1)/2$ edges. [5]

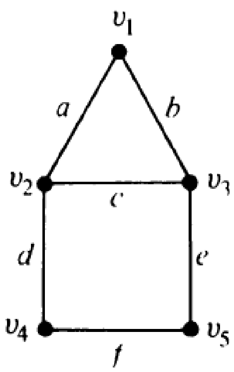
3. For the given graphs G_1 and G_2, find out the followings. [2.5 x 4=10]

   (i) $G_1 \cup G_2$

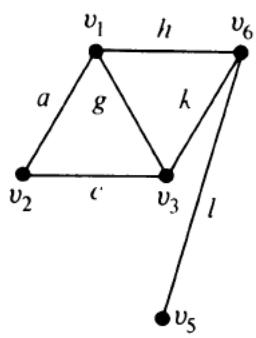
   (ii) $G_1 \cap G_2$

   (iii) $G_1 \oplus G_2$

   (iv) $G_2 - v_3$

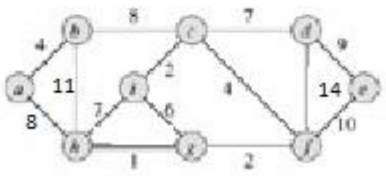


G_1



G_2

4. Determine the minimum spanning tree of the following graph and find its rank and Nullity. [10]



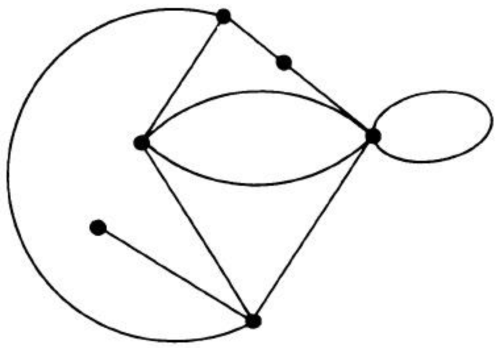
5. Prove that a connected planar graph with *n* vertices and *e* edges has *e-n+2* regions. [10]

6. (a) Prove that in any simple, connected planar graph with *f* regions, *n* vertices and *e* edges ($e > 2$), the following inequalities must hold: [5]

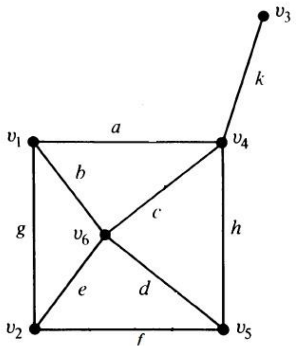
$$e \geq \frac{3}{2}f$$

$$e \leq 3n-6$$

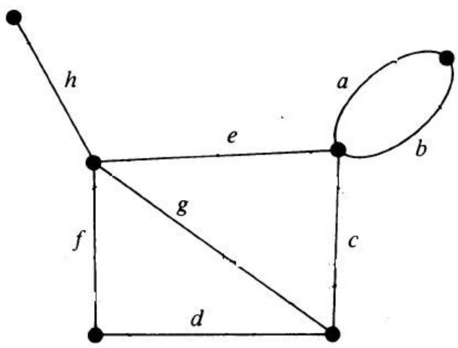
(b) Find the geometric dual of the following planar graph. [5]



7. Find the basis vectors of the circuit subspace of the following graph and show that the vectors determined form a basis. [10]



8. (a) Determine the Cut-set matrix of the following graph and mention the observations found. [6]



(b) Prove that the chromatic number of K_n *i.e.* $\chi(K_n) = n$. [4]

9. (a) Let A and B be two finite sets, with $|A| = m$ and $|B| = n$ ($n \geq m$). How many distinct one-to-one functions (mappings) can you define from set A to set B ? [5]

(b) In how many ways 10 sweets of same colour, size and shape can be distributed [5] among 6 children when there is no fair play ?