

**P.G.DEPARTMENT OF MATHEMATICS,  
UTKAL UNIVERSITY, VANI VIHAR**



**REGULATIONS AND SYLLABUS  
FOR  
M.PHIL. MATHEMATICS  
(UNDER SEMESTER SYSTEM)  
(effective from academic session 2013-2014 )**

**1. Duration of the course:** The duration of the M.Phil. Programme shall be one year consisting of two semesters.

**2. Distribution of Marks :**

**SEMESTER – I**

- Core Course : 100 marks
- Core Course : 100 marks
- Elective Course : 100 marks

**SEMESTER – II**

- Dissertation and Seminar : 200 marks

**3. Course of Study:**

- There are three papers in Semester-I out of which one of them is an elective paper and one paper in Semester-II which is Dissertation & Seminar .

**SEMESTER-I**

| PAPER     | TITLE OF THE COURSE  | MARKS |
|-----------|--|-------|
| PAPER-I   | ANALYSIS   | 100   |
| PAPER-II  | ALGEBRA, FUNCTIONAL ANALYSIS, NUMERICAL ANALYSIS                           | 100   |
| PAPER-III | ADVANCED NUMERICAL ANALYSIS/<br>GEOMETRIC FUNCTION THEORY/ SEQUENCE SPACES | 100   |

**SEMESTER-II**

| PAPER    | TITLE OF THE COURSE      | MARKS |
|----------|--------------------------|-------|
| PAPER-IV | DISSERTATION AND SEMINAR | 200   |

**4. Scheme of Examinations:**

- **Written Examination:** The examination for the papers I, II and III under Semester-I shall be held at the end of the first semester. Each paper carries a maximum of 100 marks.
- The question paper will be set by the internal examiners.
- **Dissertation:** Candidates shall submit the dissertation in the department through the supervisor before filling up the forms of second semester examination, which shall be valued by the internal examiner (supervisor) and one external examiner.

## 5. Question Paper Pattern:

### Semester-I

- Time: Four hours
- Maximum: 100 marks (One questions from each unit with internal choice)
- Each unit carries 20 marks.

## 6. Dissertation:

- **Topic:** The topic of the dissertation shall be assigned to the candidate towards the end of the Semester-I.
- **Number of copies of Dissertation:** The students should prepare three copies of the dissertation and submit the same in the department for evaluation.

## 7. Passing Minimum:

- A candidate shall be declared to have passed Semester-I examination, if he/she secures not less than 40% of the marks in each paper and 50% marks in aggregate.
- A candidate shall be declared to have passed Semester-II examination, if he/she secures not less than 60% of the marks in the paper Dissertation & Seminar (Paper-IV) .
- A candidate who has passed both the semester examinations shall be considered to have passed the M.Phil programme.

## 8. Restriction in number of chances:

- A candidate cannot resubmit a Dissertation .
- Candidates shall have to pass both the semesters within a period of two years from the date of joining the programme.
- Improvement examinations either in Semester-I or Semester-II are not allowed.

## **9. Commencement of this regulation:**

- The regulations and syllabi shall take effect from the academic year 2013–2014, that is; for those admitted to the programme during the academic year 2013 – 2014 and thereafter.
- The papers of the M.Phil. examinations will be valued as follows:

Paper-I: Internal examiner

Paper-II: Internal examiner

Paper-III(Elective paper): Internal examiner

Paper-IV: Both external and internal examiner.

## **10. The teacher's council will act as the conducting board.**

### **DETAILED SYLLABUS**

### **SEMESTER-I**

### **PAPER-I-(Analysis)**

**(Each unit carries 20 marks)**

#### **Unit-I (Real Analysis)**

Elements of metric spaces, convergence, continuity, compactness, connectedness, Weierstrass approximation theorem, Equicontinuity, Arzela-Ascoli theorem, Lebesgue measure, measurable functions, convergence in measure, Lebesgue integral, differentiation and integration.

#### **Unit-II (Complex Analysis)**

Analytic Functions, power series representation of analytic functions, Cauchy's theorem for convex regions, Cauchy's integral formula, Liouville's theorem, fundamental theorem of algebra, Open mapping theorem, Singularities Laurent series, Casorati-Weierstrass theorem, Residues, Argument principle, Maximum modulus theorem, Bilinear transformations, Multivalued analytic functions.

#### **Unit-III (Topology)**

Elements of topological space, Continuity, Convergence, Homeomorphism, Compactness, Local compactness, Locally connected, pair wise connected, Separation axioms, First and Second countability, Separable topological spaces, Product topological spaces, Quotient spaces, Tychonoff's theorem, Urysohn's lemma, Homotopy and Fundamental group.

#### Unit-IV (Functional Analysis)

Banach spaces, Hahn-Banach theorem, Open mapping and Closed graph theorem, Principle of uniform boundedness, Boundedness and continuity of linear transformations, Dual spaces, Embedding in the second dual, Hilbert spaces, Projections, Orthonormal basis, Riesz-representation theorem, Bessel's inequality, Parseval's identity, Self-adjoint operators, Normal operators.

#### Unit-V (Calculus in $\mathbb{R}^n$ )

Euclidian space  $\mathbb{R}$ , Bolzano Weierstrass theorem, Compact subsets of  $\mathbb{R}$ , Heine Borel theorem, Continuity of functions in  $\mathbb{R}$ , Differentiability of functions from  $\mathbb{R}^m$  to  $\mathbb{R}^n$ , Properties of differentials, Partial and Directional derivatives, Continuously differentiable function, Taylor series, Inverse function theorem, Implicit function theorem.

#### **BOOKS RECOMMENDED:**

1. W.Rudin : Mathematical Analysis, Mc-Graw Hill
2. G.D.Barre : Measure Theory and Integration, Willey Eastern Ltd.
3. J.B.Conway : Functions of one Complex Variable, Springer Student Edition
4. J.R.Munkres : Topology – A First Course, Prentice Hall of India
5. B.V.Limaye : Functional Analysis, Wiley Eastern Ltd.

#### **PAPER-II-(Algebra and Numerical Analysis)**

**(Each unit carries 20 marks)**

(Only problems are to be set from Unit-I, II, III)

#### Unit-I (Linear Algebra)

Symmetric groups, Alternating groups, Simple groups, Rings, Maximal ideals, Prime ideals, Integral domains, Euclidean domains, Principal ideal domains,

Unique factorization domains, Quotient fields, Finite fields, Algebra of linear transformations, Reduction of matrices to canonical forms, Inner product spaces, Orthogonality, Quadratic forms, Reduction of quadratic forms.

### Unit-II (Advanced Algebra)

Conjugate elements and class equations of finite groups, Sylow's theorem, Solvable groups, Jordan Holder theorem, Direct products, Rings, Characteristics of field extension, Elements of Galois Theory, Solvability of Radicals.

### Unit-III (Numerical Analysis)

Finite differences, Interpolation, Numerical solution of algebraic equation, Iteration, Newton-Raphson method, Solution of linear system, Direct method, Gauss elimination method, Matrix inversion, Eigen value problems, Numerical Differentiation & Integration, Numerical solution of ordinary differential equation, Iteration method, Picards method, Euler's method and improved Euler's method.

### Unit-IV & V (Advanced Functional Analysis): 40 marks

Distributions and Sobolev Spaces. Ch.1(1.1-1.6), Ch.2(2.1-2.4)

### **BOOKS RECOMMENDED:**

1. I. N. Herstein : Topics in Algebra
2. N. Jacobson : Basic Algebra
3. K. E. Atkinson : Numerical Analysis
4. Hoffman and Knice : Linear Algebra
5. Khanna & Bhambri : Basic Abstract Algebra
6. Moderson, Sen & Mallik : Abstract Algebra
7. Bhattacharjee, Jain & Nagpaul : Abstract Algebra
8. Kesavan : Functional Analysis and Applications

### **PAPER-III- (ADVANCED NUMERICAL ANALYSIS)**

#### **I. Numerical Integration (Unit-I, II, III) - 60 marks**

- i) Approximate integration over finite intervals;
- ii) Error Analysis;
- iii) Approximate of integration in two or more dimensions

## **II. Numerical solution of partial differential equation method by finite -difference method (Unit -IV & V)-40 marks**

Introduction to finite difference schemes, Convergence, Consistency and stability, Analysis of finite difference schemes order of accuracy of finite difference schemes, Stability of multistep scheme, Dissipation and dispersion, Finite difference scheme for parabolic equation, the correction-diffusion equation.

### **BOOKS RECOMMENDED:**

1. Davis and Rabinowitz - Methods of Numerical Integration (al), ch.2 (2.1-2.7, 2.11, 2.12), 4(4.1, 4.3, 4.6, 4.7), 5 (5.1-5.7).
2. John C. Strikwerda - Finite Difference Scheme and Partial Differential Equations (Wadsworth and books), Ch.1, 2, 3, 4, 5, 6.

## **OR**

### **Geometric Function Theory (Marks: 100)**

**Unit-I:** Elementary theory of Univalent functions, Area Theorem, Growth and Distortion Theorems, Coefficient Estimates, Meromorphic univalent functions, Starlike and Convex functions.

**Unit-II:** Close-to-Convex functions, Spirallike functions, Typically Real functions, Elementary properties of functions with positive real part, Functions with positive real part and real coefficients, Functions that are Starlike and Convex in one direction.

**Unit-III:** Starlike and Convex functions of order Alpha, Alpha-Convex functions and Alpha-Spirallike functions. Subordination: Basic principles, Coefficient inequalities.

**Unit-IV:** Majorization, Univalent subordinate functions, Radius of Starlikeness and Convexity. Extremal problems , Representation of Linear functionals.

**Unit-V:** Extreme points and Support points, Properties of Extremal functions, Extreme points of  $S$  and related classes.

**Books Recommended:**

- (i) P.L. Duren: Univalent Functions, Springer-Verlag, 1983.
- (ii) A.W. Goodman: Univalent Functions(Vol.I), Dover Publications, 1983

**OR**

**Sequence Spaces**

**UNIT-I: Prerequisites:**

Topological vector spaces, continuity of linear maps, types of topological vector space, results on continuity and equicontinuity, locally convex spaces, duality, properties of TVS and its dual. A condition for reflexivity, adjoints of linear maps.

**UNIT-II: Sequence spaces:**

Introduction, Basic definitions and results, duals, Topology on sequence spaces,  $K$  spaces relative to  $\alpha$ -,  $\beta$ - and  $\gamma$ -duals, spaces of the form  $\lambda_A$ ,  $\mu_A$  and  $\nu_A$ . sequence spaces: natural metrics and duals, construction of abstract BK-topologies. Dual of the space  $\lambda_N$ . Normal topology, strongest locally normal topology, Perfect, simple and symmetric spaces, Duality between perfect spaces: Duality between  $\varphi_1$  and  $\omega_\infty$ , Duality between  $l^1$  and  $l^\infty$ , Duality between  $l^p$  and  $l^q$  ( $1 < p, q < \infty$ ), Duality between  $\delta$  and  $d, \infty$ , Duality between nonperfect spaces, Duality between  $C_0$  and  $l^1$ , Duality between  $k$  and  $l^1$ , Duality between  $l^p$  ( $0 < p < 1$ ) and  $l^\infty$ , Duality between  $l^1$  and  $m_0$ , Dual of  $l^\infty$ .

**UNIT-III: Convergence of series:**

Introduction, absolute convergence, bounded multiplier and subseries convergence, interrelationships of types of convergence, weak convergence theorems, linear transportations and convergence criteria.



#### **UNIT-IV: Further developments in sequence spaces:**

Inclusion theorems for FK-spaces, preliminaries on matrix transformations, Inclusion theorems, FK-spaces containing  $C_0$ , K-spaces containing  $l^1$ ,  $b_v$  or  $b_{v_0}$ , FK-spaces containing  $l^p$  ( $1 < p < \infty$ ), FK-spaces containing  $l^\infty$ , FK-spaces containing  $l^p$  ( $0 < p < 1$ ).

#### **UNIT-V: Matrix transformations:**

Specific transformations, transformations related to simple sequence spaces, transformations related to FK-spaces, diagonal transformations, composition of transformations.

#### **Books Recommended:**

Sequence spaces and series: P.K. Kamthan & Manjul Gupta, Marcel Dekker, Inc, New York and Basel.

#### **Reference Books**

1. Sequence spaces-W.H.Ruckle, Pitman Advanced Publishing Program 1981.
2. Some sequence spaces and their geometric properties-Vakeel A. Khan, VDM publishing 2009.
3. Sequence spaces and Applications- Pawan K. Jain, Eberhat Malkowsky, Narosa Publishing House, 1999.

### **Semester-II**

- Paper-IV  
Dissertation and Seminar =200 marks